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Strapdown inertial navigation without satellite signals: error growth, IMU noise grades and visual-inertial aiding

A literature-based review of inertial error propagation, Allan-variance characterisation of IMU grades, inertial budgets for GNSS-denied intervals and visual-inertial aiding.

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Abstract

Aircraft that must navigate when satellite signals are degraded rely on an inertial measurement unit (IMU), usually aided by other sensors. This paper reviews the error mechanisms of strapdown inertial navigation, the characterisation of inertial sensors by Allan variance, the noise levels reported for IMUs of different grade, and the architecture of visual-inertial aiding. It derives the time dependence of position error caused by accelerometer bias, gyroscope bias, tilt error and white noise, and gives worked examples with generic values. Using published Allan-variance results, it compares the angle random walk of navigation-grade, tactical-grade and consumer-grade gyroscopes, which differ by more than two orders of magnitude. A measured stationary test of a low-cost strapdown system shows an average position error of 152.67 m after 60 s, almost all of it attributed to orientation error, and aiding with a magnetometer reduced it to about 5 m. Visual-inertial odometry is summarised as a complementary approach whose weaknesses (low texture, fast motion, exposure) differ from those of the inertial sensor. The sources are a mix of measurements, analytic derivations and reviews at different scales; none reports flight data from a lift-rotor aircraft, and the paper states the limits of each result.

1. Introduction and scope

An inertial navigation system (INS) computes position, velocity and orientation by integrating the outputs of accelerometers and rate gyroscopes. It provides continuous localisation even when other navigation means fail, but the sensor errors make the position error increase with time and distance travelled [3]. The selection of an inertial sensor therefore requires an analysis of how sensor errors affect position and orientation accuracy, so that the solution is cost-effective and meets the requirements [3]. When satellite navigation is unavailable, the inertial solution has to be bounded by other measurements.

The scope of this paper is the error behaviour of strapdown inertial navigation, the quantification of inertial sensor noise, a comparison of reported IMU grades, and visual-inertial aiding. It does not treat satellite receivers, terrain-referenced navigation, radio navigation, magnetic-anomaly navigation or specific products. For a general introduction to the product categories see the site’s sensors guide and its pages on IMUs and visual navigation.

2. Theory and governing equations

2.1 Error integration chains

In a strapdown system the gyroscope signals are integrated once to track orientation, and the accelerometer signals, after being projected onto the navigation axes and corrected for gravity, are integrated twice to track position [1]. A constant gyroscope bias ϵ (rad/s) inte-

grates to an angle error that grows linearly with time, $\theta(t) = \epsilon t$ [1]. A constant accelerometer bias b (m/s²), integrated twice, gives a position error that grows quadratically,

$$s(t) = \tfrac{1}{2}\, b\, t^2$$

where s is the position error (m) and t the integration time (s) [1]. For white noise of standard deviation σ on a gyroscope sampled at interval δt (s), the integrated angle error is a zero-mean random walk with standard deviation

$$\sigma_\theta(t) = \sigma\, \delta t\, \sqrt{t}$$

that grows with the square root of time [1]. Manufacturers express this as the angle random walk, $ARW = \sigma_\theta(1)$, in °/√h; after t hours the standard deviation of the angle error is $ARW\sqrt{t}$ [1]. Accelerometer white noise integrated twice gives a second-order random walk in position with standard deviation $\sigma_s(t) \approx \sigma\, t^{3/2}\sqrt{\delta t/3}$, where σ is the noise standard deviation in the units of acceleration [1]. Flicker noise, modelled as a bias random walk, produces errors that grow faster still: for the accelerometer a third-order random walk in position whose standard deviation grows as $t^{5/2}$ [1].

2.2 Propagation of a gyroscope error into position

The critical error path in nearly all inertial systems is the propagation of gyroscope errors to position, because the magnitude of gravity is usually much larger than the acceleration of the vehicle [1]. A tilt error $\delta\theta$ (rad) leaves a component of gravity $g \sin\delta\theta \approx g\,\delta\theta$ on the horizontal axes, which the system interprets as acceleration [1][2]. For a gyroscope bias b_ω (rad/s) this gives $\delta\theta = b_\omega t$ and a horizontal acceleration error $g\, b_\omega t$, so that the velocity error is $\tfrac{1}{2} g\, b_\omega t^2$ and the position error

$$p(t) = \tfrac{1}{6}\, g\, b_\omega\, t^3$$

where g is gravitational acceleration (m/s²) [2]. A gyroscope bias thus gives a second-order error in velocity and a third-order error in position, as against second order in position for an accelerometer bias [2]. The cubic law is valid for flat, non-rotating earth; the coupling of horizontal position error to gravity through the earth's curvature (Schuler feedback) gives an oscillation with period $2\pi\sqrt{R_e/g} = 84$ min (calculated here, $R_e = 6.371 \times 10^6$ m), so the law is a short-time approximation for durations that are small against this period.

2.3 Worked examples (generic)

Accelerometer bias. A bias of 100 µg (9.81×10^{-4} m/s²) gives $s = 0.05$ m after 10 s and about 490 m after 1,000 s (calculated with $g = 9.81$ m/s²; [2] gives 0.05 m and 500 m with $g = 10$ m/s²).

Gyroscope bias. A bias of 0.2 °/h (9.7×10^{-7} rad/s) gives $p = \tfrac{1}{6} g b_\omega t^3 = 1.6 \times 10^{-3}$ m after 10 s, 1.6 m after 100 s and 1.6 km after 1,000 s (calculated; [2] gives 0.0016 m and 1,600 m with $g = 10$ m/s²). The 1,000 s value is 20 % of the Schuler period, so it overstates the error and illustrates the order of magnitude only.

Tilt error. Woodman considers a tilt error of 0.05°, which projects a gravity component of $9.81 \sin(0.05^\circ) = 0.0086$ m/s² onto the horizontal axes [1]. The source states that the resulting error grows quadratically to 7.7 m after 30 s [1]. Evaluating $\tfrac{1}{2} a t^2$ with $a = 0.0086$ m/s² gives 3.9 m after 30 s (calculated here), half the value in the source, and the reason for the difference is not stated; the quadratic growth and the order of magnitude are not in doubt.

Random walk. For an angle random walk of 0.0015 °/√h (the order reported for a navigation-grade gyroscope in Section 4), the standard deviation of the angle error is 0.0015° after 1 h and 0.0030° after 4 h; for 0.5 °/√h (consumer grade) it is 0.5° after 1 h and 0.2° after 10 min (calculated from $ARW\sqrt{t}$).

2.4 Dependence on motion

The simple relations above assume little motion. Borodacz and Szczepański derived analytic formulas for the orientation and position errors of an unaided strapdown system for piecewise-constant motion, including bias, random walk and scale-factor errors, and showed that the errors depend on the manoeuvres and not only on the navigation duration [3]. In particular, rotation does not change the orientation error about the axis of rotation, and continuous rotation increases the error in that axis because of a linearly increasing scale-factor error [3]. The formulas assume a flat, inertial, orthogonal navigation frame [3]. As long as the specific force is zero, the orientation and position errors are independent of each other [3]. The authors conclude that the formulas allow more precise specification of requirements for selecting measurement systems [3].

3. Design and sizing methods

3.1 Allan variance

The Allan variance is the method of IEEE Std 952-1997 for identifying noise terms of inertial sensors from static data [2]. Static output is averaged in clusters of length T, and the Allan variance is half the mean square difference between adjacent cluster averages,

$$\sigma^2(T) = \tfrac{1}{2} \left\langle \left[\Omega_{next}(T) - \Omega_k(T) \right]^2 \right\rangle$$

where $\Omega_k(T)$ is the average of cluster k (in the unit of the sensor output) [2]. For each noise term the Allan deviation has a characteristic slope on a log-log plot against T$$\sigma(T) | Slope | Reading of the coefficient | |---|---|---| | Quantisation | $\sqrt{3}\, Q_z / T$ | −1 | at $T = \sqrt{3}$ | | Angle (velocity) random walk | $Q / \sqrt{T}$ | −1/2 | at $T = 1$ | | Bias instability | plateau of $0.664\, B$ | 0 | flat region | | Rate random walk | $K \sqrt{T/3}$ | +1/2 | at $T = 3$ |

Here Q_z, Q, B and K are the coefficients of quantisation noise, random walk, bias instability and rate random walk [2]. Hou reports that the same coefficients obtained from the power spectral density and from the Allan variance agree, and that the Allan procedure is

simpler [2]. The bias-instability plateau may be hidden by the influence of other noise terms, and the quality of the estimate depends on the number of independent clusters [2].

3.2 Aiding with other sensors

Sensor fusion and constraints specific to the application reduce inertial drift [1]. In the stationary test of Woodman, aiding with magnetometers reduced the average position error after 60 s from over 150 m to about 5 m, and the author concludes that a MEMS-based INS cannot be built to give sub-metre accuracy for more than one minute of operation with the technology of the time [1]. This is a result for one low-cost device at rest; it does not describe a moving vehicle.

3.3 Sizing of the inertial budget

The relations of Section 2 can be inverted to obtain the sensor quality that a mission requires. For a permissible position error p_{max} (m) after an outage of duration t (s), an accelerometer bias must satisfy $b \leq 2\,p_{max}/t^2$ and a gyroscope bias, through the tilt path of Section 2.2, $b_\omega \leq 6\,p_{max}/(g\,t^3)$. Both are bounds on the residual bias after calibration and in-run estimation, not on the specification of the unprocessed sensor.

Worked example (generic). For $p_{max} = 100$ m after $t = 600$ s: $b \leq 2 \times 100/600^2 = 5.6 \times 10^{-4}$ m/s² (57 µg), and $b_\omega \leq 6 \times 100/(9.81 \times 600^3) = 2.8 \times 10^{-7}$ rad/s, or 0.058 °/h (calculated here). The duration of 600 s is small against the Schuler period of 84 min, so the cubic law is acceptable. Compared with the consumer-grade bias instability of 11.7–16.8 °/h in Table 1, the gyroscope bound is lower by a factor of about 200 to 290; the angle random walk adds a random component on top of the bias bound. The example shows that for outages of several minutes the gyroscope, not the accelerometer, sets the grade, in agreement with [1], and that the bias instability of consumer-grade gyroscopes cannot meet such a budget without aiding. Aiding by the methods of Sections 3.2 and 3.4 is therefore the means of bounding the error, with the inertial unit bridging the interval between aiding updates.

3.4 Visual-inertial odometry

Visual-inertial odometry (VIO) estimates the pose and velocity of an agent using only cameras and IMUs [4]. The two sensors are complementary: cameras are precise during slow motion and give rich information, but they have a limited output rate (about 100 Hz), suffer from scale ambiguity in a monocular configuration and are not robust in scenes with low texture, high-speed motion (motion blur) or high dynamic range; an IMU is scene-independent and has a high output rate (about 1,000 Hz), but its biases make the estimate drift quickly [4]. The measurement model is $\omega = {}^{I}\omega + b_g + n_g$ and $a = R_{IW}({}^{W}a - {}^{W}g) + b_a + n_a$, where b are biases and n white noises (for low-cost IMUs scale factors and axis misalignment may have to be added) [4]. A single IMU makes metric scale and gravity observable, which a monocular camera alone does not [4]. VIO approaches are loosely coupled when visual and inertial estimates are computed separately and then fused, or tightly coupled when raw measurements are fused jointly; tightly coupled methods are more accurate [4]. Estimators are filters or smoothers: filters estimate

only the latest state and are efficient, but they suffer from linearisation errors and inconsistency, and the problem has four unobservable directions (global position and rotation about gravity), while fixed-lag smoothing is generally more accurate and more computationally expensive [4].

4. Parameters and typical ranges

Table 1 collects noise coefficients identified by Allan variance in [2] for three gyroscope grades from static tests of two hours, with the quantities in the units of the source, and the relations of Section 2.

Parameter	Navigation grade	Tactical grade	Consumer (MEMS) grade	Source
Gyroscope angle random walk (°/√h)	0.0015–0.0019	0.075–0.094	0.49–0.51	[2]
Gyroscope bias instability (°/h)	not identified	not identified	11.7–16.8	[2]
Accelerometer velocity random walk (m/s/√h)	—	—	0.016–0.017	[2]
Accelerometer bias instability (m/s/h)	not identified	not identified	4.1–4.4	[2]

Further reported quantities:

Parameter	Value	Type	Source
Average position error of a low-cost strapdown INS at rest, 60 s	152.67 m (vertical component 1.76 m)	Measured, 500 runs	[1]
Same system aided by magnetometers, 60 s	about 5 m	Measured	[1]
Orientation error (tilt) to produce 0.0086 m/s² of horizontal bias	0.05°	Calculated	Section 2.3
Accelerometer bias 100 µg: position error at 10 s / 1,000 s	0.05 m / about 490 m	Calculated	Section 2.3
Gyroscope bias 0.2 °/h: position error at 100 s	1.6 m	Calculated	Section 2.3
Period of the Schuler oscillation	84 min	Calculated	Section 2.2
Camera output rate / IMU output rate	about 100 Hz / about 1,000 Hz	Typical, stated	[4]
Unobservable directions of VIO	4 (global position, rotation about gravity)	Theory	[4]

For comparison, if the consumer-grade bias instability of 13.6 °/h were an uncompensated constant bias, the cubic law would give a position error of 23 m after 60 s, against 0.34 m for 0.2 °/h (calculated); the measured 152.67 m of [1] is for a different device and is dominated by white noise on the gyroscope signals [1].

5. Failure modes, testing and qualification

5.1 Identification of the dominant error

A simulation in [1] removed accelerometer and gyroscope noise selectively and showed that nearly all of the drift of the low-cost system was due to orientation error, that white noise on the gyroscopes contributed most, and that accelerometer noise was significant only in the first seconds [1]. The result confirms the error path of Section 2.2 and means that qualification of an inertial unit should include its gyroscope noise and bias behaviour over the mission duration [1][2].

5.2 Test methods

The Allan-variance test is a static test of long duration; the sources used data of two hours to seven days per IMU [2]. Noise terms that were not identified in a given data set may need longer test data to show whether they exist [2]. Bias and scale-factor errors are estimated by calibration: an accelerometer bias is difficult to measure because gravity appears as a bias unless the orientation is known precisely, which is why calibration routines use a turntable whose orientation is controlled accurately [1]. Bias shows a dependence on temperature, and a residual temperature effect is integrated into the orientation error [1]. For visual-inertial systems the failure conditions are those of the camera: low texture, fast motion and extreme exposure [4].

5.3 Regulatory context

The EASA special condition for small-category VTOL-capable aircraft (SC-VTOL-02, Issue 2, June 2024) requires that, for the equipment and systems it covers, each catastrophic failure condition be extremely improbable and not result from a single failure, each hazardous failure condition be extremely remote and each major failure condition be remote (VTOL.2510(a)) [5]. For Category Enhanced it also requires provisions for in-service monitoring of equipment and systems whose failure may have hazardous or catastrophic consequences (VTOL.2510(c)) [5]. How the loss of satellite navigation and the drift of an inertial solution map to failure conditions depends on the operation and the aircraft architecture and is not specified in the text reviewed; the performance requirement for navigation in a GNSS-denied condition is a design decision supported by the error models of Section 2.

6. Open problems and limitations

1. **Environment.** The error models assume small motion, flat earth and constant biases; the analytic derivation of [3] assumes a flat, inertial navigation frame, and the Schuler feedback limits the validity of the cubic law to durations short against 84 min.
2. **Static tests.** The Allan-variance results and the 152.67 m drift were obtained with the sensors at rest in a laboratory; vibration, temperature changes and manoeuvres, which add scale-factor and misalignment errors [3], are not represented.
3. **Source inconsistency.** A numerical example in [1] (7.7 m after 30 s for a 0.05° tilt) differs by a factor of two from the quadratic law calculated here (3.9 m); the source does not explain it.
4. **Sensor generations.** The grades in Table 1 come from sensors characterised in 2004 [2] and a low-cost device tested in 2007 [1]; the performance of current devices is not reported in these sources and must be taken from current test data.
5. **Visual-inertial limits.** The VIO review is a descriptive survey and gives no numerical accuracy figures for aircraft at altitude or in featureless terrain; its stated weaknesses (texture, motion blur, dynamic range) apply [4].
6. **Navigation requirement.** The sources reviewed do not define a position accuracy or duration required of a lift-rotor aircraft in a GNSS-denied condition.

7. Conclusions

- Position error caused by a gyroscope bias grows with the cube of time and by an accelerometer bias with the square of time; for 0.2 °/h the cubic law gives 1.6 m after 100 s, and for 100 µg the quadratic law gives about 490 m after 1,000 s (calculated).
- White noise on the gyroscope signals was the dominant cause of the drift of a low-cost strapdown INS at rest, with an average position error of 152.67 m after 60 s, reduced to about 5 m by magnetometer aiding [1].
- Allan variance separates quantisation, random walk, bias instability and rate random walk by the slope of $\sigma(T)$; reported gyroscope angle random walk spans from about 0.0015 °/√h (navigation grade) through about 0.09 °/√h (tactical grade) to about 0.5 °/√h (consumer grade) [2].
- Orientation and position errors depend on the manoeuvres as well as on the duration [3].
- Visual-inertial odometry combines two sensors with complementary failure conditions, makes metric scale observable and is more accurate when tightly coupled [4].
- No reviewed source defines a required accuracy for navigation without satellite signals in a VTOL-capable aircraft; the safety requirements of SC-VTOL-02 apply to the failure conditions of the installed equipment [5].

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